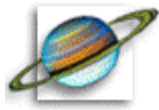


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CHAPTER 3 ASTRONOMICAL PHENOMENA

3.1 Sunrise, Sunset, and Twilight



Introduction

This document calculates, for latitudes between 65 degrees North and 65 degrees South and any date in the latter half of the 20th century, the times of **sunrise**, **sunset**, and civil, nautical, and astronomical twilight. The calculated times refer to locations with a clear horizon and a standard refraction correction for normal meteorological conditions. The calculated times are accurate to within 2 minutes; higher accuracy is seldom justified since local meteorological conditions will cause the actual time to vary.

Twilight is caused by the scattering of sunlight by the upper layers of the Earth's atmosphere. Astronomical twilight begins at sunset (ends at sunrise) and ends (begins) when the Sun's center is 18 degrees below the horizon. Civil twilight begins (ends) when the Sun's center is 6 degrees below the horizon; nautical twilight begins (ends) when the Sun's center is 12 degrees below the horizon.

The method of the *Almanac for Computers* (United States Naval Observatory) is used to calculate the times.

The definitions for sunrise, sunset, and twilight for an observer at sea level are given in terms of the **zenith distance** z ($\pi/2$ - altitude) of the center of the Sun:

Sunrise and Sunset	90 deg 50 arc min
Civil Twilight	96
Nautical Twilight	102
Astronomical Twilight	108

Sunrise or Sunset is defined for a geocentric altitude of -50 arc minutes; 34 arc minutes is to correct for atmospheric refraction and 16 arc minutes is to correct for the Sun's semidiameter.

For an observer at a height h meters above sea level the following correction should be added to the zenith distances above:

$$2.12h^{1/2} \text{ arc minutes}$$

The following formulae are used to compute the phenomena:

$$M = 0.985600t - 3.289 \text{ degrees}$$

$$\lambda = M + 1.916\sin M + .020\sin 2M + 282.634 \text{ degrees}$$

$$\tan \alpha = 0.91746\tan \lambda \quad \sin \delta = 0.39782\sin \lambda$$

$$\cos H = (\cos z - \sin \delta \sin \phi) / (\cos \delta \cos \phi)$$

$$T = H + \alpha - 0.065710t - 6.622 \text{ hours}$$

$$UT = T + L$$

where

- M = Sun's **mean anomaly**
- t = approximate time of phenomenon in days since 0 January, 0^h UT
- λ = Sun's true **longitude**
- α = Sun's **right ascension**
- δ = Sun's **declination**
- H = Sun's local **hour angle**
- T = local mean time of phenomenon
- ϕ = **latitude** of observer
- L = **longitude** of observer (east is negative; west is positive) in hours
- UT = **Universal Time** of phenomenon

Method

Using the above formulae, the procedure from the Almanac for Computers is as follows:

1. Calculate an initial value for t from

$$t = N + (6^h + L)/24 \quad (\text{for morning phenomenon})$$

$$t = N + (18^h + L)/24 \quad (\text{for evening phenomenon})$$

where N is the day of the year.

2. Calculate M and λ .
3. Solve for α , noting that α is in the same quadrant as L .
4. Solve for δ .
5. Solve for H ; the correct quadrant for H is given by the following rules:

$$\text{rising phenomenon:} \quad H = 360 - \arccos x$$

$$\text{setting phenomenon:} \quad H = \arccos x$$

6. Calculate T .
7. Calculate UT. To convert to the local time see the standard time conversions in **Section 1.16**.

Enter Data

Year: Y := 1988

Month: MM := 1

Day of the Month: D := 15

Latitude: $(\phi_{\text{deg}} \ \phi_{\text{min}} \ \phi_{\text{sec}}) := (31 \ 41 \ 18)$

Longitude: $(L_{\text{deg}} \ L_{\text{min}} \ L_{\text{sec}}) := (110 \ 53 \ 6)$

Observer's height
above horizon
(meters): h := 2608

Observer's location: MMT Observatory,
Arizona

Calculations

Intermediate Variables

Convert to decimal form:

The functions necessary to convert between sexagesimal and decimal form are defined past the right margin of the page.

$$L := \frac{\text{sexa2dec}(L_{\text{deg}}, L_{\text{min}}, L_{\text{sec}})}{15}$$

$$\phi := \text{sexa2dec}(\phi_{\text{deg}}, \phi_{\text{min}}, \phi_{\text{sec}})$$

The day of the year N:

First, calculate the **Julian Date** for the calendar date at 0^h UT:

$$y := \text{if}(\text{MM} > 2, Y, Y - 1) \quad m := \text{if}(\text{MM} > 2, \text{MM}, \text{MM} + 12)$$

$$\text{jd} := \text{floor}(365.25 \cdot y) + \text{floor}[30.6001 \cdot (m + 1)] \dots \\ + D + 1720994.5$$

$$a := \text{floor}\left(\frac{y}{100}\right) \quad b := 2 - a + \text{floor}\left(\frac{a}{4}\right)$$

$$\text{JD}_t := \text{jd} + b \quad \text{JD}_t = 2447175.5$$

Then calculate the **Julian Date** for the calendar date on 0 January, 0^h UT for the given year:

$$\text{MM} := 1 \quad D := 0$$

$$y := \text{if}(\text{MM} > 2, Y, Y - 1)$$

$$m := \text{if}(\text{MM} > 2, \text{MM}, \text{MM} + 12)$$

$$\text{jd} := \text{floor}(365.25 \cdot y) + \text{floor}[30.6001 \cdot (m + 1)] \dots \\ + D + 1720994.5$$

$$a := \text{floor}\left(\frac{y}{100}\right) \quad b := 2 - a + \text{floor}\left(\frac{a}{4}\right)$$

$$\text{JD}_0 := \text{jd} + b \quad \text{JD}_0 = 2447160.5$$

$$N := \text{JD}_t - \text{JD}_0 \quad N = 15$$

Define an initial value for t, the approximate time of the phenomena:

$$t_m := N + \frac{6 + L}{24} \quad (\text{for morning phenomena})$$

$$t_e := N + \frac{18 + L}{24} \quad (\text{for evening phenomena})$$

The **zenith distances** for the phenomena:

$$z_0 := \left(90 + \frac{50 + 2.12 \cdot \sqrt{h}}{60} \right) \cdot \text{degrees} \quad \text{for Sunrise or Sunset}$$

$$z_1 := 96 + 2.12 \cdot \frac{\sqrt{h}}{60} \quad \text{for civil twilight}$$

$$z_2 := 102 + 2.12 \cdot \frac{\sqrt{h}}{60} \quad \text{for nautical twilight}$$

$$z_3 := 108 + 2.12 \cdot \frac{\sqrt{h}}{60} \quad \text{for astronomical twilight}$$

Calculate the Morning Phenomenon

$$M := 0.985600 \cdot t_m - 3.289$$

$$\lambda := M + 1.916 \cdot \sin(M \cdot \text{deg}) + 0.020 \cdot \sin(2 \cdot M \cdot \text{deg}) + 282.634$$

$$\lambda := \text{mod}(\lambda, 360)$$

$$\alpha := \frac{180}{\pi} \cdot \frac{\text{angle}(\cos(\lambda \cdot \text{deg}), 0.91746 \cdot \sin(\lambda \cdot \text{deg}))}{15}$$

$$\delta := \left(\frac{180}{\pi} \right) \cdot \text{asin}(0.39782 \cdot \sin(\lambda \cdot \text{deg}))$$

$$i := 0 \dots 3$$

$$x_i := \frac{\cos(z_i \cdot \text{deg}) - \sin(\delta \cdot \text{deg}) \cdot \sin(\phi \cdot \text{deg})}{\cos(\delta \cdot \text{deg}) \cdot \cos(\phi \cdot \text{deg})}$$

$$H_i := \frac{360 - \left(\frac{180}{\pi} \right) \cdot \text{acos}(x_i)}{15}$$

$$T_i := H_i + \alpha - 0.065710 \cdot t_m - 6.622$$

Convert the time T to a range between 0 and 24^h:

$$T_i := \text{mod}(T_i, 24)$$

$$T_i := \text{if}(T_i < 0, T_i + 24, T_i)$$

Calculate UT of the phenomenon. If UT is greater than 24^h, the phenomenon occurs on the following day, Greenwich time. If UT is negative, the phenomenon occurs on the previous day, Greenwich time.

$$UT_i := T_i + L$$

Calculate Evening Phenomenon

$$M := 0.985600 \cdot t_e - 3.289$$

$$\lambda := M + 1.916 \cdot \sin(M \cdot \text{deg}) + 0.020 \cdot \sin(2 \cdot M \cdot \text{deg}) + 282.634$$

$$\lambda := \text{mod}(\lambda, 360)$$

$$\alpha := \frac{180}{\pi} \cdot \frac{\text{angle}(\cos(\lambda \cdot \text{deg}), 0.91746 \cdot \sin(\lambda \cdot \text{deg}))}{15}$$

$$\delta := \left(\frac{180}{\pi} \right) \cdot \text{asin}(0.39782 \cdot \sin(\lambda \cdot \text{deg}))$$

$$i := 0..3$$

$$x_i := \frac{\cos(z_i \cdot \text{deg}) - \sin(\delta \cdot \text{deg}) \cdot \sin(\phi \cdot \text{deg})}{\cos(\delta \cdot \text{deg}) \cdot \cos(\phi \cdot \text{deg})}$$

$$H_i := \left(\frac{180}{\pi \cdot 15} \right) \cdot \text{acos}(x_i)$$

$$T_i := H_i + \alpha - 0.065710 \cdot t_e - 6.622$$

Convert the time T to a range between 0 and 24^h:

$$T_i := \text{mod}(T_i, 24)$$

$$T_i := \text{if}(T_i < 0, T_i + 24, T_i)$$

Calculate UT of the phenomenon. If UT is greater than 24^h, the phenomenon occurs on the following day, Greenwich time. If UT is negative, the phenomenon occurs on the previous day, Greenwich time.

$$UT_i := T_i + L$$

Results

The Morning Phenomena (converting to sexagesimal notation)

Sunrise:

$$UT_{\text{rise}} := \left[\text{floor}(UT_0) \quad (UT_0 - \text{floor}(UT_0)) \cdot 60 \right]$$

$$UT_{\text{rise}} = (24 \ 52)$$

Civil twilight:

$$UT_{\text{civil}} := \left[\text{floor}(UT_1) \quad (UT_1 - \text{floor}(UT_1)) \cdot 60 \right]$$

$$UT_{\text{civil}} = (25 \ 18)$$

Nautical twilight:

$$UT_{\text{nautical}} := \left[\text{floor}(UT_2) \quad (UT_2 - \text{floor}(UT_2)) \cdot 60 \right]$$

$$UT_{\text{nautical}} = (25 \ 48)$$

Astronomical twilight:

$$UT_{\text{astro}} := \left[\text{floor}(UT_3) \quad (UT_3 - \text{floor}(UT_3)) \cdot 60 \right]$$

$$UT_{\text{astro}} = (26 \ 17)$$

Note: To convert to Eastern Standard Time (for example),
subtract 5 hours from the above times.

The Evening Phenomena
(converting to sexagesimal notation)

Sunset:

$$UT_{\text{set}} := \left[\text{floor}(UT_0) \quad (UT_0 - \text{floor}(UT_0)) \cdot 60 \right]$$

$$UT_{\text{set}} = (24 \quad 52)$$

Civil twilight:

$$UT_{\text{civil}} := \left[\text{floor}(UT_1) \quad (UT_1 - \text{floor}(UT_1)) \cdot 60 \right]$$

$$UT_{\text{civil}} = (25 \quad 18)$$

Nautical twilight:

$$UT_{\text{nautical}} := \left[\text{floor}(UT_2) \quad (UT_2 - \text{floor}(UT_2)) \cdot 60 \right]$$

$$UT_{\text{nautical}} = (25 \quad 48)$$

Astronomical twilight:

$$UT_{\text{astro}} := \left[\text{floor}(UT_3) \quad (UT_3 - \text{floor}(UT_3)) \cdot 60 \right]$$

$$UT_{\text{astro}} = (26 \quad 17)$$

Note: To convert to Eastern Standard Time (for example),
subtract 5 hours from the above times.

Accuracy

In order to provide the user with an indication of the accuracy of the above method, we have calculated the times of Sunrise and Sunset for the year 1993 and compared these times with those times provided by the Astronomical Almanac. The times were calculated for a latitude of 42 degrees and at the Greenwich meridian.

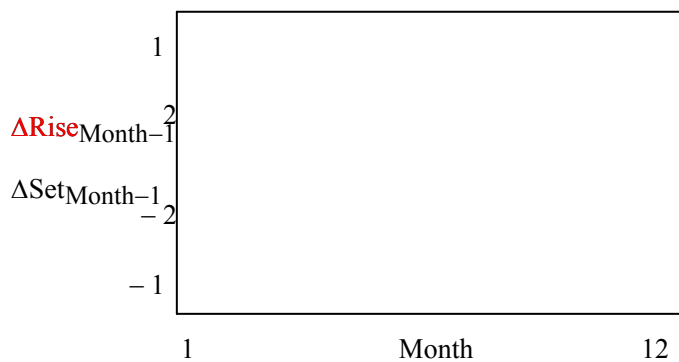
The differences in Sunrise $\Delta\text{Rise} = \text{Rise}_{\text{almanac}} - \text{Rise}_{\text{handbook}}$ and Sunset $\Delta\text{Set} = \text{Set}_{\text{almanac}} - \text{Set}_{\text{handbook}}$ in minutes for the beginning of each of the twelve months of 1993 are:

$$\Delta\text{Rise} := \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}^T$$

$$\Delta\text{Set} := \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}^T$$

A plot of the above differences:

Month := 1..12



Sexagesimal (angles or time) to decimal

The conversion function is given by

$$\text{sexa2dec}(a, b, c) := a + \frac{b}{60} + \frac{c}{3600}$$

where

a = integral number of degrees or hours

b = integral number of arc minutes or minutes of time

c = integral number of arc seconds or seconds of time

Unit Definitions:

hours $\equiv$ 1

degrees $\equiv$ 1

Decimal (angles or time) to sexagesimal

We must first define some intermediate functions:

$$\text{hr_deg}(d_h) := \text{floor}(|d_h|)$$

$$\text{min}(d_h) := \text{floor}((|d_h| - \text{hr_deg}(d_h)) \cdot 60)$$

$$\text{sec}(d_h) := \left(|d_h| - \text{hr_deg}(d_h) - \frac{\text{min}(d_h)}{60} \right) \cdot 3600$$

where d_h is the angle or time in decimal notation.

The conversion function is then given by

$$\text{dec2sexa}(d_h) := (\text{hr_deg}(d_h) \ \text{min}(d_h) \ \text{sec}(d_h)) \cdot \frac{d_h}{|d_h|}$$